

DÉVELOPPEMENTS LIMITÉS

Formule de Taylor-Young

Si f est de classe \mathcal{C}^n au voisinage de a , alors f admet le $DL_n(a)$ suivant :

$$\begin{aligned} f(x) &= f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \dots + \frac{f^{(n)}(a)}{n!}(x-a)^n + o((x-a)^n) \\ &= \sum_{k=0}^n \frac{f^{(k)}(a)}{k!}(x-a)^k + o((x-a)^n) \end{aligned}$$

Puissance, logarithme, exponentielle

$$\begin{aligned} \frac{1}{1-x} &= \sum_{k=0}^n x^k + o(x^n) &= 1 + x + x^2 + \dots + x^n + o(x^n) \\ \frac{1}{1+x} &= \sum_{k=0}^n (-1)^k x^k + o(x^n) &= 1 - x + x^2 - x^3 + x^4 \dots + (-1)^n x^n + o(x^n) \\ \ln(1+x) &= \sum_{k=1}^n (-1)^{k-1} \frac{x^k}{k} + o(x^n) &= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots + (-1)^{n-1} \frac{x^n}{n} + o(x^n) \\ e^x &= \sum_{k=0}^n \frac{x^k}{k!} + o(x^n) &= 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots + \frac{x^n}{n!} + o(x^n) \\ f(x) &= \sum_{k=0}^n \binom{\alpha}{k} x^k + o(x^n) &= 1 + \alpha x + \frac{\alpha(\alpha-1)x^2}{2} + \dots + \frac{\alpha(\alpha-1)(\alpha-2)\dots(\alpha-n+1)x^n}{n!} + o(x^n) \end{aligned}$$

Fonctions circulaires

$$\begin{aligned} \sin(x) &= \sum_{k=0}^n (-1)^k \frac{x^{2k+1}}{(2k+1)!} + o(x^{2n+1}) &= x - \frac{x^3}{6} + \frac{x^5}{120} - \dots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+1}) \\ \cos(x) &= \sum_{k=0}^n (-1)^k \frac{x^{2k}}{(2k)!} + o(x^{2n}) &= 1 - \frac{x^2}{2} + \frac{x^4}{24} - \dots + (-1)^n \frac{x^{2n}}{(2n)!} + o(x^{2n}) \\ \arctan(x) &= \sum_{k=0}^n (-1)^k \frac{x^{2k+1}}{2k+1} + o(x^{2n+1}) &= x - \frac{x^3}{3} + \frac{x^5}{5} - \dots + (-1)^n \frac{x^{2n+1}}{2n+1} + o(x^{2n+1}) \\ \tan(x) &= x + \frac{x^3}{3} + o(x^3) \end{aligned}$$

Fonctions hyperboliques

$$\begin{aligned} \operatorname{sh}(x) &= \sum_{k=0}^n \frac{x^{2k+1}}{(2k+1)!} + o(x^{2n+1}) &= x + \frac{x^3}{6} + \frac{x^5}{120} + \dots + \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+1}) \\ \operatorname{ch}(x) &= \sum_{k=0}^n \frac{x^{2k}}{(2k)!} + o(x^{2n}) &= 1 + \frac{x^2}{2} + \frac{x^4}{24} + \dots + \frac{x^{2n}}{(2n)!} + o(x^{2n}) \end{aligned}$$